Differentiation and Approximation Calculus of a Single Variable X. Du

- **Newton's Method**
  - Used to approximate the zero of a function $f(x)$.
  - An algorithm using the recursive formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, where x_n is your current estimate of the zero and x_{n+1} is your next approximation of the zero. Iteration will get you closer and closer to the actual zero.
  - Draw graph to understand how this recursive formula is established.
  - Sometimes fails in cases when:
    - The function is not differentiable on the closed interval between the initial estimate and the zero.
    - There are horizontal asymptotes and the initial estimate is to the side of the relative extrema beyond which the function approaches a limit without crossing the x-axis.
    - The estimate is at a point where $f'(x) = 0$.
    - The graph of the function turns without touching the x-axis.
- **Linear Approximation** (first-order Taylor polynomial)
  - Also known as **differentials**
  - Develop a function $y = f(x)$ to model what is being approximated.
  - Pick a point easy to evaluate (x_o, y_o) very close to what you're attempting to approximate. The farther the point you pick, the less accurate your approximation will be.
  - Evaluate the derivative of your function at (x_o, y_o).
  - Let the difference between your actual value and your estimated point x_o be *dx*.
  - $\frac{dy}{dx} = f'(x) \Rightarrow dy = f'(x)dx$ $\quad y = y_o + dy$ (*dy* is the change in *y*)
- **Euler's Method**
  - Utilizes linear approximation with iteration to approximate a function represented by a differential equation.
  - Uses very small steps for *dx* known as a step. $dy = f'(x)dx$
  - Requires a differential equation and a point.
  - Use a table! Have a column for *x*, *y*, $f'(x)$, and *dy*.
- **Uncertainty** propagation in laboratory settings
  - Also uses differentials and linear approximation
  - Let $y = f(x)$ and $\frac{dy}{dx} = f'(x)$ or $dy = f'(x)dx$ at a point. For an uncertainty in *x* denoted *dx*, there is a corresponding uncertainty in *y* found in *dy*.

Further notes:

- Newton's method is a technique used by many computers to find zeros of functions by iteration, as computers can make a repeated calculation very quickly in very little time.
- Some scientists may use Gaussian error propagation methods (not discussed here)